Constraint Satisfaction and Optimization

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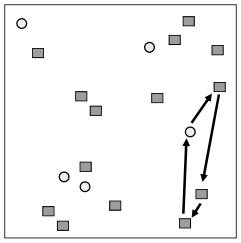
BT's Workforce Scheduling

BT has many jobs to be done in UK every day. It has to schedule a large number of teams to serve these jobs, subject to time, skill and other constraints. Saving of 0.5% could mean Millions of Pounds per year. Guided Local Search achieved the best results in one of BT's challenge problems.



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What is Constraint Satisfaction?

- Constraint satisfaction is a decision problem
  - You are given a number of decisions to make
  - For each decision, you have limited choices
  - Decisions constrain each other
- Your task is to make those decisions without violating any of the constraints
  - Sometimes some solutions are preferred to others

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Constrained Optimization

- Sometimes you want the "best" solution
- If so, you have a (constrained) *optimisation* problem
  - E.g. one may want to minimize transportation cost
- Sometimes you have multiple objectives
- If so, you have a multi-objective optimization problem
  - E.g. one may want to minimize cost and maximize customer satisfaction

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Airline Applications

British Airways uses constraint satisfaction to schedule aircraft to serve their flights (with a system called Eclipse) and to allocate stands for aircraft at airports (with ILOG Solver).

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Transportation

Constraint satisfaction is one of the core technologies in transportation optimization. For example Guided Local Search was used in ILOG Solver's vehicle routing package, Dispatcher. Cairo/Line schedule for collective transportation.

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Image Processing

Microsoft AutoCollage uses constraint satisfaction (among other techniques) to blend photos together.

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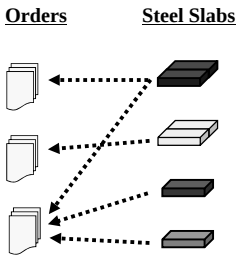
CP in Steel Industry

IBM helped Korea's biggest steel manufacturer to apply constraint technology to schedule its production to meet orders. This include the allocation of existing stocks to orders, subject to various constraints, such as size, quality and colour, in order to minimize waste and cost.



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The Unsung Heroes

- People recognize brand names and end products
- When looking at a car, few would ask:
  - Who built the engines?
  - Who made the nuts and bolts?
  - What metal did they use?

Why constraint satisfaction?

- It is ubiquitous (seen everywhere)
  - Mainly logistics, scheduling, resources allocation
- Specialized methods available
- Now multi-million Pounds business
- Scientific challenges:
  - Combinatorial explosion (fundamental problem)
  - Modelling (engineering problem)

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Combinatorial Explosion

- Suppose one needs to schedule 20 technicians to serve 40 jobs
  - Very small problem in practice
- But there are 20^{40} ($\approx 10^{52}$) combinations
  - Trying all combinations will take years in the fastest computers today
- As the number of jobs increases, the number of combinations increases exponentially.

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Constraint Technology Matters

- Combinatorial explosion is a core problem in computation science
  - It is the very problem that makes chess hard and passwords hard to crack
- Naïve algorithms have no chance
- Some algorithms are cleverer than others
  - They can find solutions quicker
  - Or they can find better solutions

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Summary, Constraint Satisfaction

- Focus on (limited choices) decision problem
- Constraint satisfaction is everywhere
- Clever algorithms have been developed
  - One tries to exploit features of the problems
- This is a successful multi-£M business
  - Though most people don't know about it
- Constraint technology can't be ignored

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Use The Force

- Constraints are the very reason why we have a hard problem to solve
- But to the trained eyes, constraints tell us how to solve the problem!
- Constraints could help in:
  - Look ahead search
  - Learning at dead ends
  - Use heuristics

Technical Introduction

A Glimpse of Constraint Satisfaction

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Sample Constraint Applications

- Resources Allocation
  - Staff rostering, timetabling
  - British Telecom's work force scheduling
  - British Airway's flight scheduling
- Transportation
  - ***ILOG Dispatcher*** for vehicle routing
  - Train and bus scheduling
  - Collective transportation
- Industrial Scheduling
  - Car sequencing
  - Steel industry
- Other Applications
  - Microsoft AutoCollage
  - Staff empowerment
  - Satellite scheduling
  - Radio links assignment (military & mobiles)
  - Phone network routing
  - Container port

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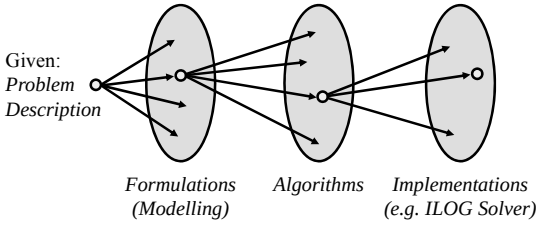
Constraint Programming



- Areas: modelling, algorithms, algorithm mapping, monitoring, packages

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Constraints Handling Techniques

- Constraint Propagation
  - Constraints tell you the consequence of each decision
  - They could help to eliminate certain choices; or
  - Help to reduce the remaining problem to an easier one
- Dependency Directed Backtracking and Learning
  - When one gets stuck,
  - Constraints could help identifying culprit decisions
- Heuristic search
  - Constraints could tell you what decisions are more critical,
  - And which choices are more promising
  - Essex contribution: Guided Local Search

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Constraint Techniques Overview

- **Problem Reduction**
- **Systematic (Exhaustive) Search**
  - **Brute-force Search**
  - **Lookahead Search**
  - **More Lookahead**
  - **Failure Handling at Dead-ends**
- **Stochastic methods**
  - **Motivation: Combinatorial Explosion**
  - **Local Search**

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Maintaining Arc-Consistency

- Variables: x, y, z
- Domains: {1, 2, 3, 4}
- Constraints:

$x < y$; $y < z$

x	1	2	~~3~~	~~4~~
y	~~1~~	2	3	~~4~~
z	~~1~~	~~2~~	3	4

• x<y means <x,4> is not supported by y and <y,1> is not supported by x

• y<z means <y,4> is not supported by z and <z,1> & <z,2> is not supported by y

• Re-check x<y would delete <x,3> as now (with <y,4> gone) it has no support from y

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Backtracking Search In The 8-Queens Problem

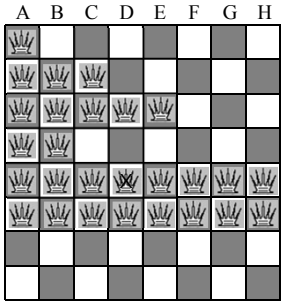
Complete search, till solution found, or "no solution" is concluded

- Place one queen per row

Strategies at dead-ends:
- Learning "no goods"
- dependency directed backtracking

column

Backtrack at dead-ends



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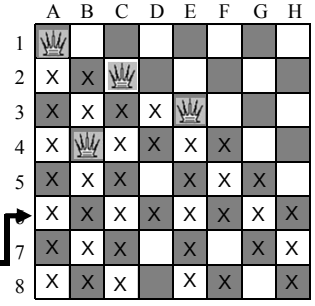
Forward Checking Search

- Problem reduction
  - a major technique
- Combined with search methods
- Reduce domain of future variables
- Detect dead-ends
  - To backtrack early

Dead-end detected after Queen 4 – no legal space for row 6, backtrack…



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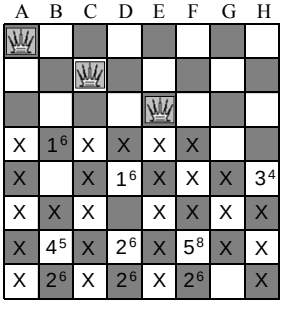
Arc-Consistency Lookahead

- 8-Queens Problem
- Three queens have been placed
- Maintaining **AC**
- Result: dead-end found in row 7
- Backtrack required – typically remove Queen 3



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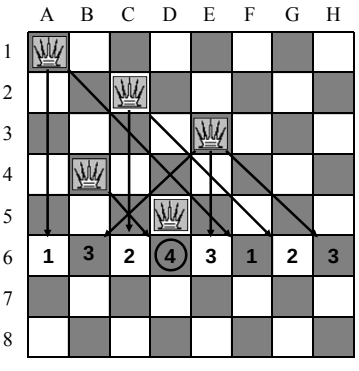
Back Jumping

- Jump to the latest culprit

Recorded earliest conflict

Identify the latest culprit, which is 4

Undo queens 5 and 4, continue



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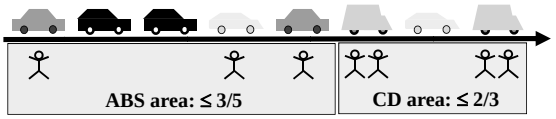
Car Sequencing Problem

Options					
ABS	×	√	√	×	
CD	×	×	√	√	
…					Total:
Production:	30	30	20	40	120



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Combinatorial Explosion of the Car Sequencing Problem

- Schedule 30 cars:
  - Search space: 30 factorial ≅ $\mathbf{10^{32}}$ leaf nodes
- Generously allow:
  - Explore one in every $\mathbf{10^{10}}$ leaf nodes!
  - Examine $\mathbf{10^{10}}$ nodes per second!
- Problem takes over **32 thousand years** to solve!!!
  - $\mathbf{10^{32} \div 10^{10} \div 10^{10} \div 60 \div 60 \div 24 \div 365 \cong 31{,}710}$
- How to contain ***combinatorial explosion***?

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Heuristic Repair, Example

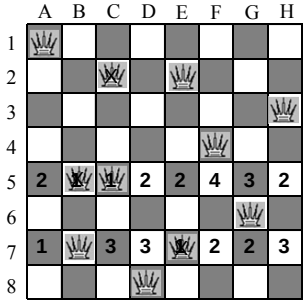
- Start with random assignments
- C2 attacks G6
  D8 attacks E7
- Randomly pick one, say, E7, to repair

Count number of conflicts in each square

Randomly pick a square with least attacks, say, B7

Repeat repair

Solution found



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Constraint Handling Techniques Summary

- Problem Reduction
  - Reduce the problem to an easier problem
- Exhaustive Search Techniques, e.g.:
  - Look ahead in order to detect dead-ends
  - Backtrack intelligently at dead-ends
- Stochastic Search
  - Handle problems which are too big to solve
  - Normally through "repairing" solutions

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