

Constraint Satisfaction in Discrete Optimisation

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Abstract

Inspired by constraint satisfaction as well as operations research, Guided Local Search (GLS) and Fast Local Search (FLS) are novel stochastic constraint satisfaction and optimisation methods. They have been applied to a number of problems and achieved remarkable results. For example, they achieve results as good as, if not better than the state-of-the-art algorithms in the travelling salesman problem. In this paper, we shall outline these algorithms and describe some of their discrete optimisation applications.

1. Introduction

Constraint satisfaction [Tsang 1993, Freuder & Mackworth 1994, Marriott & Stuckey 1998] is a very general problem that is required in many real life problems. Due to its generality, much research effort has been spent in this area in recent years. This has led to technological break-through as well as commercial exploitation. Sound commercially-available systems have been built, e.g. see ILOG Solver [Puget 1995], CHIP [Simonis 1995], ECLiPSe [Lever et al. 1995] and Prolog IV [Colmerauer 1990]. Constraint programming is now a multi-million Pounds business; see [Cras 1993] [Wallace 1996] [Zweben & Fox 1994] for some of their applications.

Most real life constraint optimisation problems are too complex for systematic search methods. In recent years, stochastic methods have received great attention [Freuder *et al.* 1995]. This paper describes the results of a research programme which has led to successful applications of stochastic constraint satisfaction techniques to optimisation.

2. Constraint satisfaction related to discrete optimisation

Many problems involve constraint satisfaction. A constraint satisfaction problem (CSP) comprises three elements:

$$(Z, D, C)$$

where Z is a finite set of variables; D is a function that maps every variable x in Z to a set of objects (of any type), which is called the *domain* of x . Most research in constraint satisfaction deal with discrete and finite domains. C is a set of constraints, which may take any form, which restricts the values that variables may take simultaneously. The task is to assign one value to each variable satisfying all the constraints [Tsang 1993]. Constraint satisfaction is a general problem which has been applied to a wide variety of problems [Freuder & Mackworth 1994, Wallace 1996].

In many constraint satisfaction problems, some solutions are "better" than others, where "better" is defined by some domain-dependent objective functions. The task in such problems is to find the optimal (minimum or maximum) solution. In other problems, constraints are classified as *hard* and *soft* constraints. Hard constraints are not to be violated in any case. Soft constraints can be violated at certain costs. In some problems, assigning different values to different variables involve different utilities. The task is to minimise the cost of maximise utilities of the solution. Finite constraint satisfaction problems that involve optimisation are basically discrete optimisation problems traditionally studied in Operations Research. There has been cross-fertilisation between the two fields. The work described in this paper started from constraint satisfaction and benefited from ideas in operations research.

3. Background: Hill Climbing

3.1. Basic Principles of Hill Climbing

Due to their combinatorial explosion nature, many real life constraint optimisation problems are hard to solve using complete methods such as *branch & bound* [Hall 1971, Reingold *et.*

al. 1977]. One way to contain the combinatorial explosion problem is to sacrifice completeness. Some of the best known methods that use this strategy are local search methods, the basic form of which are often referred to as hill climbing.

To perform hill climbing, one must define the following:

- (a) a representation for candidate solutions;
- (b) an objective function: given any candidate solution, this function returns a numerical value. The problem is seen as an optimisation problem according to this objective function (which is to be minimized or maximized);
- (c) a neighbourhood function that maps every candidate solution x (often called a *state*) to a set of other candidate solutions (which are called *neighbours* of x).

Hill climbing works as follows: starting from a candidate solution, which may be randomly or heuristically generated, the search moves to a neighbour which is 'better' according to the objective function (in a minimization problem, a better neighbour is one which is mapped to a lower value by the objective function). The search terminates if no better neighbour can be found, or resources run out. The whole process can be repeated from different starting points.

One of the main problems with hill climbing is that it may settle in local optima — states which are better than all their neighbours but not necessarily the best possible solution. To overcome that, methods such as *Simulated Annealing* [Aarts & Korst 1989, Davis 1987, Otten & van Ginneken 1977] and *Tabu Search* [Glover *et. al.* 1989, 1990, 1993] have been proposed.

3.2. Example of hill climbing: the travelling salesman problem

The *travelling salesman problem* (TSP) is a well-known optimisation problem. Given a number of cities and the distances between them, the task is to find a tour that visits all cities. The objective is to find the tour with the shortest distance travelled.

One way to hill climb on a TSP with n cities is to represent a candidate solution by a

sequence of n variables where variable i represents the i -th city to be visited in the tour. For example, a tour through 10 cities may be:

1 4 5 8 2 7 6 9 3 10

The objective function is the total distances to be travelled in a given tour. One simple but reasonably effective neighbourhood function is *2-Opt*. Effectively what it does is to pick a subsequence and reverse the order of the cities. For example, applying 2-Opt to the above tour, may get:

1 4 5 9 6 7 2 8 3 10

In other words, the subsequence 8-2-7-6-9 is reversed. This tour qualifies to be accepted by hill climbing if the total travelling distance incurred is shorter than that in the previous tour. When many neighbours are 'better' than the previous tour, one may choose a random one. Alternatively, heuristics may be applied to select among the qualified neighbours. For example the *steepest descent* heuristic will select the neighbour that incurs the least travelling distance.

4. Fast Local Search (FLS)

One factor which limits the efficiency of a hill climbing algorithm is the size of the neighbourhood. If there are many neighbours to consider, then if the search takes many steps to reach a local optima, and/or each evaluation of the objective function requires a nontrivial amount of computation, then the search could be very costly. Bentley [1992] presented the *approximate 2-Opt* method to reduce the neighbourhood of 2-Opt in the TSP. We generalised this method to a method that we call *Fast Local Search* (FLS). The intention is to, guided by heuristics, ignore neighbours that are unlikely to lead to fruitful hill-climbs in order to improve the efficiency of a search.

Here we shall use the TSP to show how FLS can be applied to the 2-Opt neighbourhood

function. An *activation bit* is associated to each transition position in the tour (e.g. transition between the third and fourth positions in the tour). All activation bits are switched on at start. Only positions with an *on* activation bit will be examined to see if it can make an improvement move. If no improvement is possible, then this bit is switched off. It will only be switched on again under two conditions:

- (1) if a 2-Opt step (initiated by another position) is made which inverts a subsequence which ends in this position. For example, if the subsequence between the fourth and the sixth cities were reversed, then the activation bit for both transitions third-to-fourth and sixth-to-seventh will be switched on.
- (2) if this transition is a feature that is penalised (to be explained when we introduce GLS later).

To generalise this to neighbourhood functions other than the 2-Opt, one may define features for candidate solutions. Selecting such features in an application is not difficult because the objective function is often made up of a number of features in the candidate solutions.

By reducing the size of the neighbourhood, one may significantly reduce the amount of computation involved in each hill climbing step. The hope is to enable more hill climbing steps in a fixed amount of time. The danger of ignoring certain neighbours is that some improvements may be missed. The hope is that the gain out-weighs the loss. We found that FLS combined extremely well with GLS.

5. Guided Local Search (GLS)

Guided local search (GLS) is a metaheuristic algorithm which aim is (like simulated annealing and tabu search) to help hill climbing to escape local optima. The basic idea is to augment the objective function with penalties, which directs the search away from local optimum. GLS was built upon our experience in a connectionist method called GENET (which stands for "Generic Network") [Wang & Tsang 1991, Tsang & Wang 1992, Davenport et. al. 1994] as well as penalty ideas from operations research [Koopman 1957,

Stone 1983, Luenberger 1984].

GLS is an algorithm for modifying the behaviour of hill climbing. To apply GLS, one has to define *features* for the candidate solutions. For example, in the travelling salesman problem, a feature could be "whether the candidate tour travels immediately from city A to city B".

GLS associates *costs* and *penalty* to each feature. The costs should normally take their values from the objective function. For example, in the travelling salesman problem, the cost of the above feature is the distance between cities A and B. The penalties are initialized to 0 and will only be increased when the local search reaches local optimum. This will be elaborated below.

Given an objective function g that maps every candidate solution s to a numerical value, we define a function h which will be used by hill climbing (replacing g).

$$h(s) = g(s) + \lambda \times \sum(p_i \times I_i(s)) \quad (1)$$

where s is a candidate solution, λ is a parameter to the GLS algorithm, i ranges over the features, p_i is the penalty for feature i (all p_i 's are initialized to 0) and I_i is an indication of whether s exhibits feature i :

$$I_i(s) = 1 \text{ if } s \text{ exhibits feature } i; 0 \text{ otherwise.} \quad (2)$$

When the local search settles on a local optimum, the penalty of some of the features associated to this local optimum is increased (to be explained below). This has the effect of changing the objective function (which defines the "*landscape*" of the local search) and driving the search towards other candidate solutions. The key to the effectiveness of GLS is in the way that penalties are imposed. It is worth pointing out that a slight variation in the way that penalties are managed could make all the difference to the effectiveness of a local search.

Our intention is to penalize "bad features", or features which "matter most", when a local search settles in a local optima. The feature which has high cost affects the overall cost more.

Another factor which should be considered is the current penalty value of that feature. We define the utility of penalizing feature i , $util_i$, under a local optimum s^* , as follows:

$$util_i(s^*) = I_i(s^*) \times c_i / (1 + p_i) \quad (3)$$

where c_i is the cost and p_i is the current penalty value of feature i . In other words, if a feature is not exhibited in the local optimum, then the utility of penalizing it is 0. The higher the cost of this feature (c_i), the greater the utility of penalizing it. Besides, the more times that it has been penalized, the lower the utility of penalizing it again.

In a local optimum, the feature(s) with the greatest $util$ value will be *penalized*. This is done by incrementing its penalty value by 1:

$$p_i = p_i + 1 \quad (4)$$

By taking cost and the current penalty into consideration in selecting the feature to penalize, we are distributing the search effort in the search space. Candidate solutions which exhibit “good features”, i.e. features involving lower cost, will be given more effort in the search, but penalties help to prevent all effort be directed to the best features. The idea of distributing search effort, which plays an important role in the success of GLS, is borrowed from Operations Research, e.g. see Koopman [1957] and Stone [1983]. Following we shall describe the general GLS procedure:

Procedure **GLS** (input: an objective function g ; a local search strategy L ; features and their costs; parameter λ)

1. Generate a starting candidate solution randomly or heuristically;
1. Initialize all the penalty values (p_i) to 0;
2. Repeat the following until a termination condition (e.g. a maximum number of iterations or time limit) has been reached:
 - 3.1. Perform local search (using L) according to the function h (which is g plus the penalty values, as defined in (1) above) until a local optimum M has been reached;
 - 3.2. For each feature i which is exhibited in M compute $util_i = c_i / (1 + p_i)$
 - 3.3. Penalize every feature i such that $util_i$ is maximum: $p_i = p_i + 1$;
4. Return the best candidate solution found so far according to the objective function g .

6. Applications of GLS and FLS in discrete optimisation problems

GLS and FLS have been applied to a number of discrete optimisation problems. They have been applied to *radio link frequency assignment problem* (RLFAP) [Bouju *et. al.* 1995] and British Telecom's *work force scheduling* problem (WFS) [Baker 1993, Azarmi & Abdul-Hameed 1995]. In the RLFAP, the task is to assign available frequencies to communication channels satisfying constraints that prevent interference. In some RLFAPs, the goal is to minimize the number of frequencies used. GLS+FLS reported the best results when it was published [Voudouris & Tsang 1996].

In British Telecom's WFS, the task is to assign technicians from various bases to serve various jobs, which may include customer requests and repairs, at various locations. Customer requirements and working hours restrict the times that certain jobs can be served by certain technicians. The objective is to minimize a function which take into consideration the travelling cost, overtime cost and unserved jobs. In the WFS, GLS+FLS still holds the best published results in the benchmark problem known to the authors [Tsang & Voudouris 1997].

The most significant results of GLS and FLS are probably in their application to the travelling salesman problem (TSP). The Lin-Kernighan algorithm (LK) is a specialised algorithm for TSP that has long been perceived as the champion of this problem [Lin & Kernighan 1973, Martin & Otto 1996]. We tested GLS+FLS+2Opt against LK in a set of benchmark problems from the public TSP library [Reinelt 1991]. Given the same amount of time (we tested 5 cpu minutes and 30 cpu minutes on a DEC Alpha 3000/600), GLS+FLS+2Opt found better results than LK in average. GLS+FLS+2Opt also outperformed the best Simulated Annealing [Johnson], Tabu Search [Knox 1994] and Genetic Algorithm [Freisleben & Merz 1996] methods reported on TSP so far. One must be cautious when interpreting such empirical results as they could be affected by many factors, including implementation issues [Hooker 1995]. But given that the TSP is an extensively studied problem, it takes something special for an algorithm to out-perform the champions. It must be

emphasized that LK is specialized for TSP but GLS and FLS are much simpler general-purpose algorithms. Details of GLS+FLS applied to the TSP can be found in [Voudouris & Tsang 1998].

GLS has also been applied to general function optimisation problems to illustrate that artificial features can be defined for problems in which the objective function suggests no obvious features. Results show that, as expected, GLS spreads its search effort across solution candidates depending on their quality (as measured by the objective function). Besides, GLS consistently found solutions in a landscape with many local sub-optimals [Voudouris & Tsang 1995].

7. Extension of GLS

Genetic algorithms (GAs) borrow their ideas from evolution [Holland 1975, Davis 1987, 1991, Goldberg 1989]. The idea is to maintain a set of candidate solutions. Individuals are given different chances to produce offspring depending on their "*fitness*". Applied to optimisation, *fitness* is measured by the objective function. GAs have been applied to constraint satisfaction [Eiben *et. al.* 1994, Ruttkay *et. al.* 1995] and demonstrated promising in discrete optimisation [Warwick & Tsang 1994, 1995].

Guided Genetic Algorithm (GGA) is a hybrid of GA and GLS. It is found to be robust, in the sense that solutions found by GGA were as good as GLS (not surprising, as GGA is built upon GLS), but within a narrower range. GGA has been applied to the Processors Configuration Problem [Lau & Tsang 1997, 1998b], General Assignment Problem [Lau & Tsang 1998a] and the Radio Length Frequency Assignment Problem [Lau & Tsang, to appear] with excellent results. Details of GGA and its applications will be reported in another occasion.

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